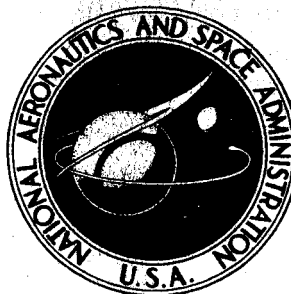


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RONDVU/PRESTO

PROGRAM FOR RENDEZVOUS MISSION TRAJECTORY OPTIMIZATION

*by Richard C. Rosenbaum, Robert E. Willwerth, Jr.
and Richard L. Moll*

*Prepared by
LOCKHEED MISSILES & SPACE COMPANY
Sunnyvale, Calif.
for Langley Research Center*

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RONDVU/PRESTO

PROGRAM FOR RENDEZVOUS MISSION TRAJECTORY OPTIMIZATION

ABSTRACT

This report presents documentation for the addition of an orbital rendezvous option to the trajectory optimization program known as PRESTO VERSION DELTA which was developed for NASA Langley Research Center. This option provides a program capability of determining boost to rendezvous trajectories with a moving target of specified ephemeris such that the time and location of the rendezvous will be optimized simultaneously with the boost trajectory shaping. Rendezvous occurs when the booster and the target have identical position and velocity vectors.

RONDVU/PRESTO
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1. FOREWORD

This report was prepared by the Aerospace Sciences Laboratory of the Lockheed Missiles and Space Company, Palo Alto, California. It represents the final documentation for the addition to the digital computer Program for Rapid Earth-to-Space Trajectory Optimization (PRESTO)* of an orbital rendezvous option (RONDVU), developed as Task II of NASA Contract NAS 1-5255 for the Langley Research Center. A FORTRAN source listing and symbolic deck included with the master copy of this report complete the program documentation. Dr. R. C. Rosenbaum was responsible for program development. Dr. Rosenbaum, Mr. R. E. Willwerth and Mr. R. L. Moll were responsible for the engineering development, and the major portion of the computer programming was done by Mr. Moll. The work performed was under the cognizance of Mr. R. L. Nelson and Mr. D. I. Kepler of the LMSC Aerospace Sciences Laboratory.

* Published as NASA CR-158 by Robert E. Willwerth, Jr., Richard C. Rosenbaum, and Wong Chuck.

2. INTRODUCTION

The ability of PRESTO to evaluate the performance of multi-stage boost systems has been extended to include orbital rendezvous missions.

Modifications to the basic PRESTO program, plus the addition of the RONDVU subroutine, have resulted in the capability of boosting to rendezvous with a moving target of specified ephemeris such that the time and location of the rendezvous will be optimized simultaneously with the boost trajectory shaping. Rendezvous occurs when the booster and the target have identical position and velocity vectors.

Solution of the rendezvous problem for the maximum payload capability of a given booster requires the use of a nonplanar trajectory, an adjustable launch time and initial launch vector, and an optimized thrust vector orientation program. If a coast stage is permitted, the payload is also affected by the duration of the coast, the minimum allowable coast altitude, and the duration of thrust cycles on either side of the coast.

The PRESTO/RONDVU digital computer program solves this performance problem by the simultaneous optimization of pitch and yaw tilt programs, terminal constraint combinations (implicit), intermediate state variable constraints, and a variety of adjustable parameters with a closed-loop steepest descent optimization routine.

A general description of how the RONDVU subroutine fits into the

basic PRESTO program organization is presented in Section 3 along with two block flow diagrams. New symbols are defined in Section 4. Section 5 serves as a manual for operating the program, including input and output discussions and comments on trouble shooting. All equations used in the RONDVU subroutine are listed in Section 6. Section 7 provides a discussion of the theoretical aspects required to modify PRESTO for the rendezvous problem, and indicates where changes were made in existing subroutines.

3. PROGRAM ORGANIZATION

Sequencing of Trajectory Iterations

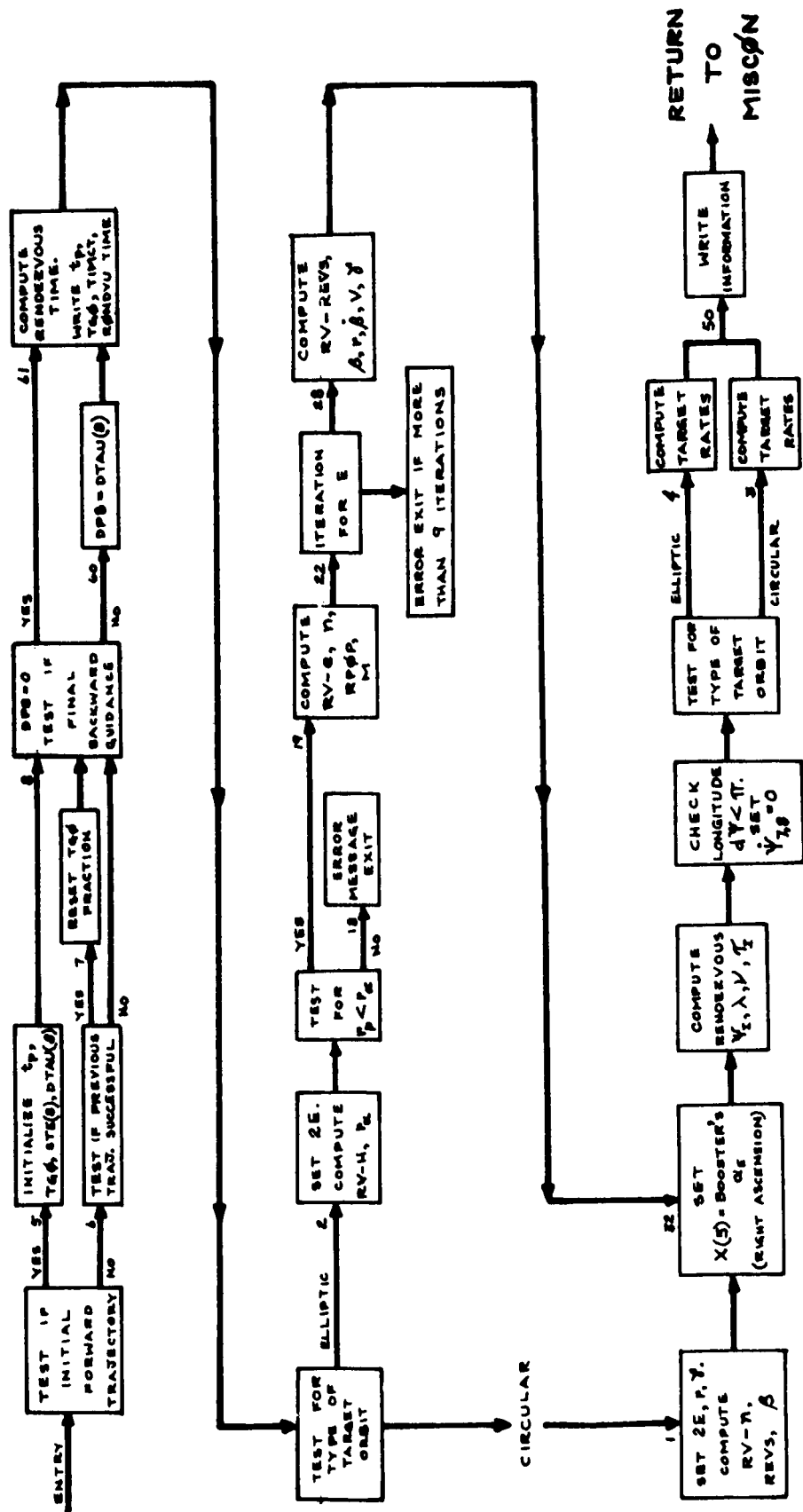
The sequence of trajectory iterations and the computational flow among subroutines for the rendezvous mission is identical to that in PRESTO except for the addition of the RONDVU subroutine. At the end of each forward trajectory, the PRESTO subroutine MISCON calls RONDVU, which computes the target's position and velocity vector from an ephemeris that is input in conventional orbit elements. The target's state variables are used as the terminal constraints for the boost trajectory. The time derivatives of these variables are also evaluated by RONDVU for use in the control equations as discussed in Section 7.

RONDVU Flow Charts

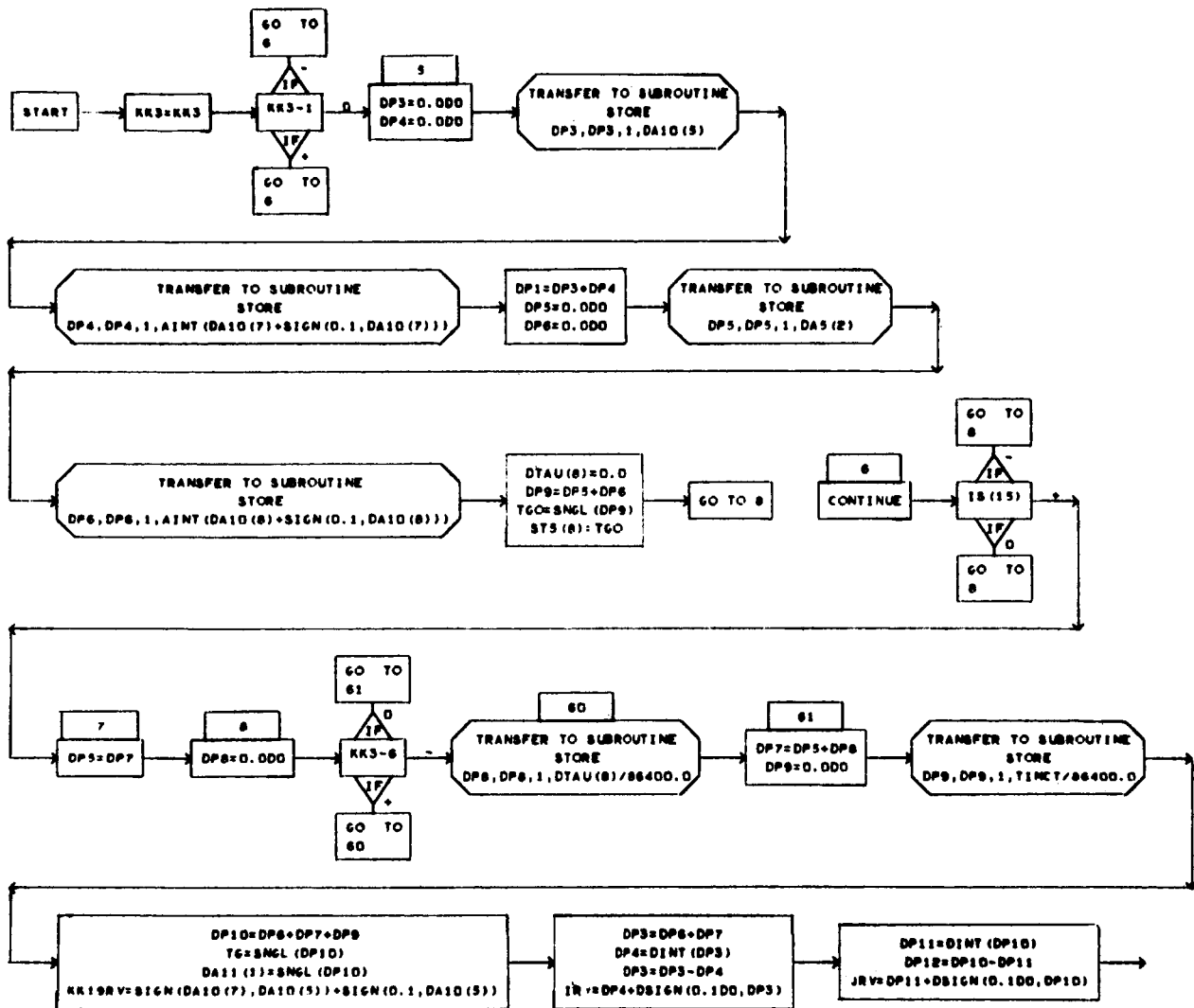
The function of the RONDVU subroutine is to correctly set the launch and rendezvous times, and convert the target's orbit elements to the instantaneous trajectory variables existing at the rendezvous time, and compute their corresponding rates at this time. Branches are available for both circular and elliptic orbits. RONDVU is called from the MISCON subroutine.

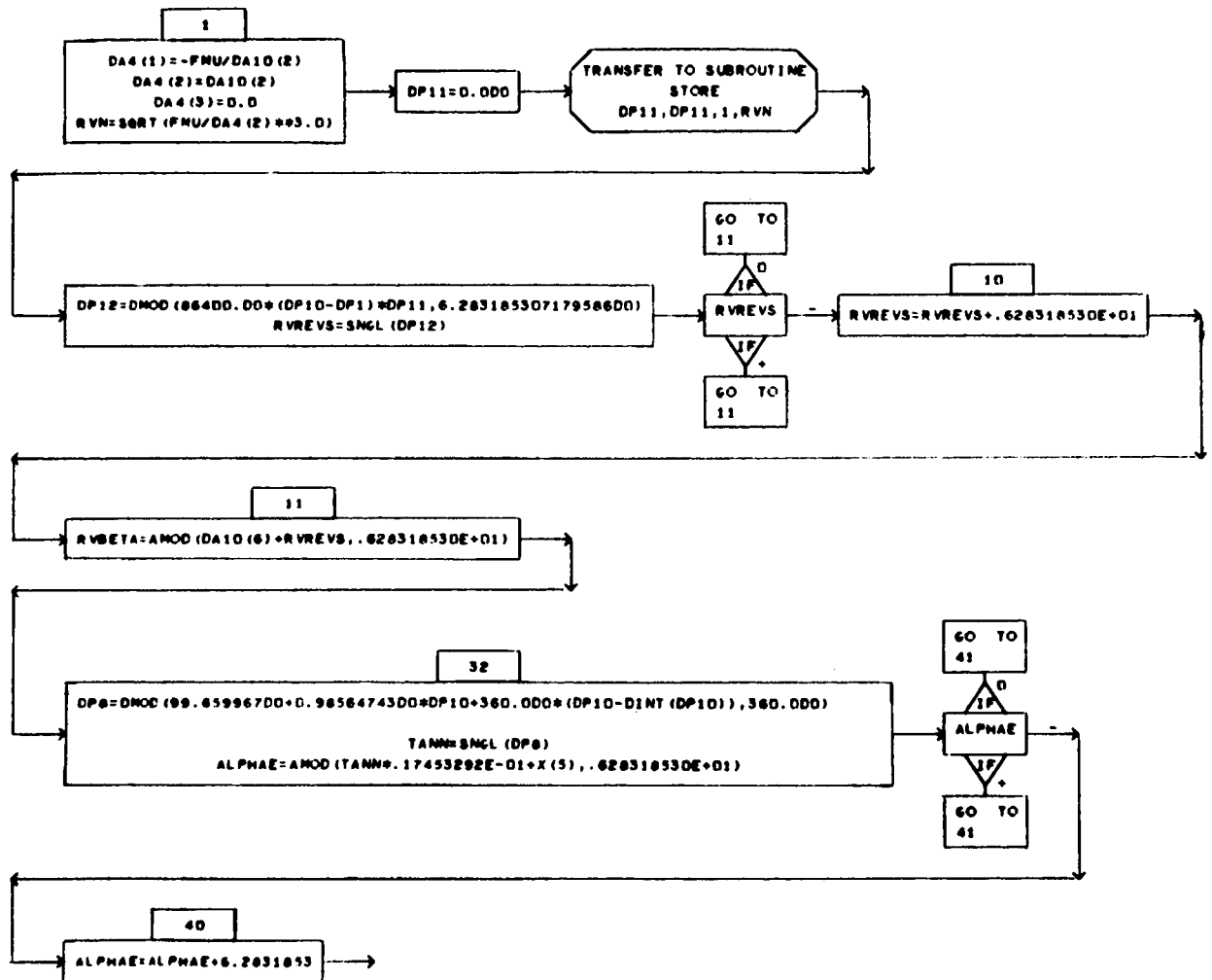
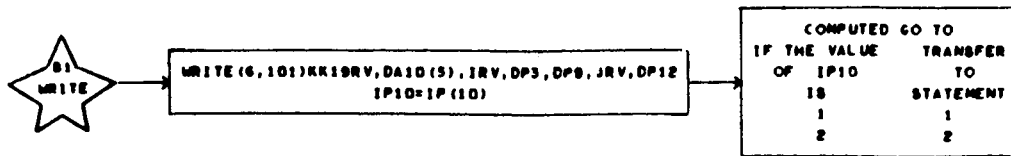
A conceptual flow chart, showing the general computational flow, is presented on the next page. Following that is a detailed flow chart which shows all the tests, logic, and equations used. These two block flow diagrams are complimentary, since the second represents exact documentation.

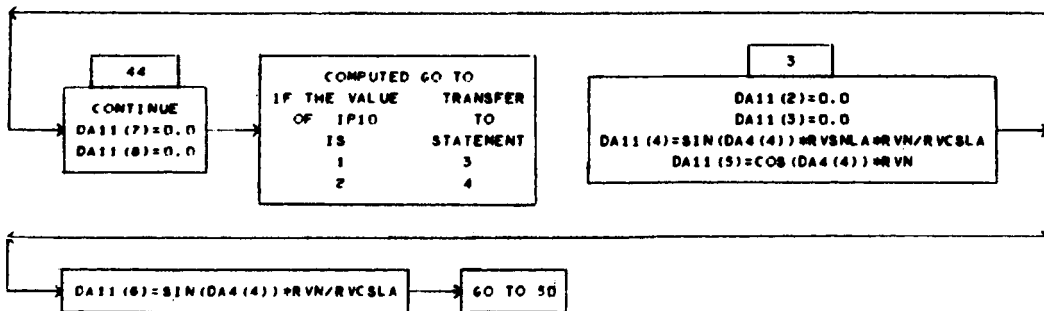
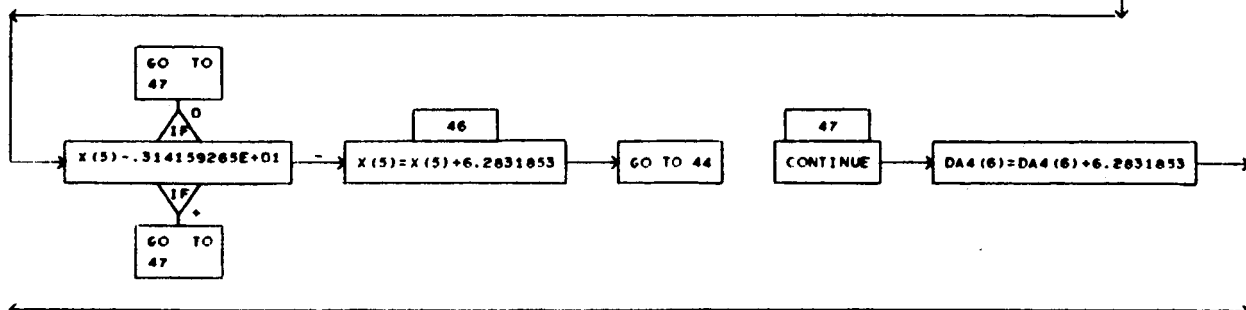
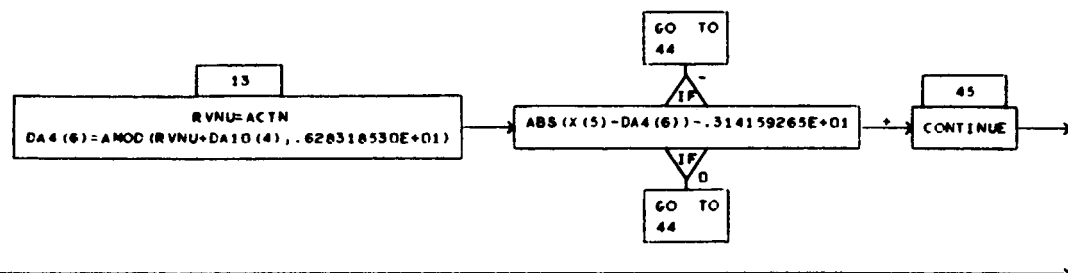
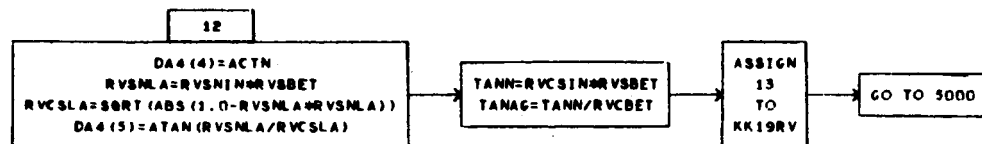
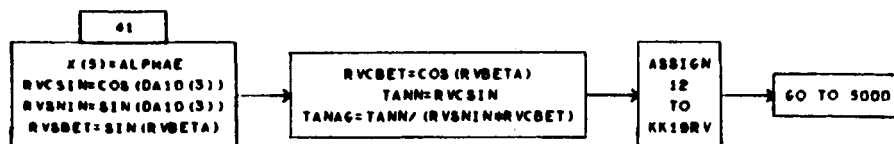
RØNDVU SUBROUTINE

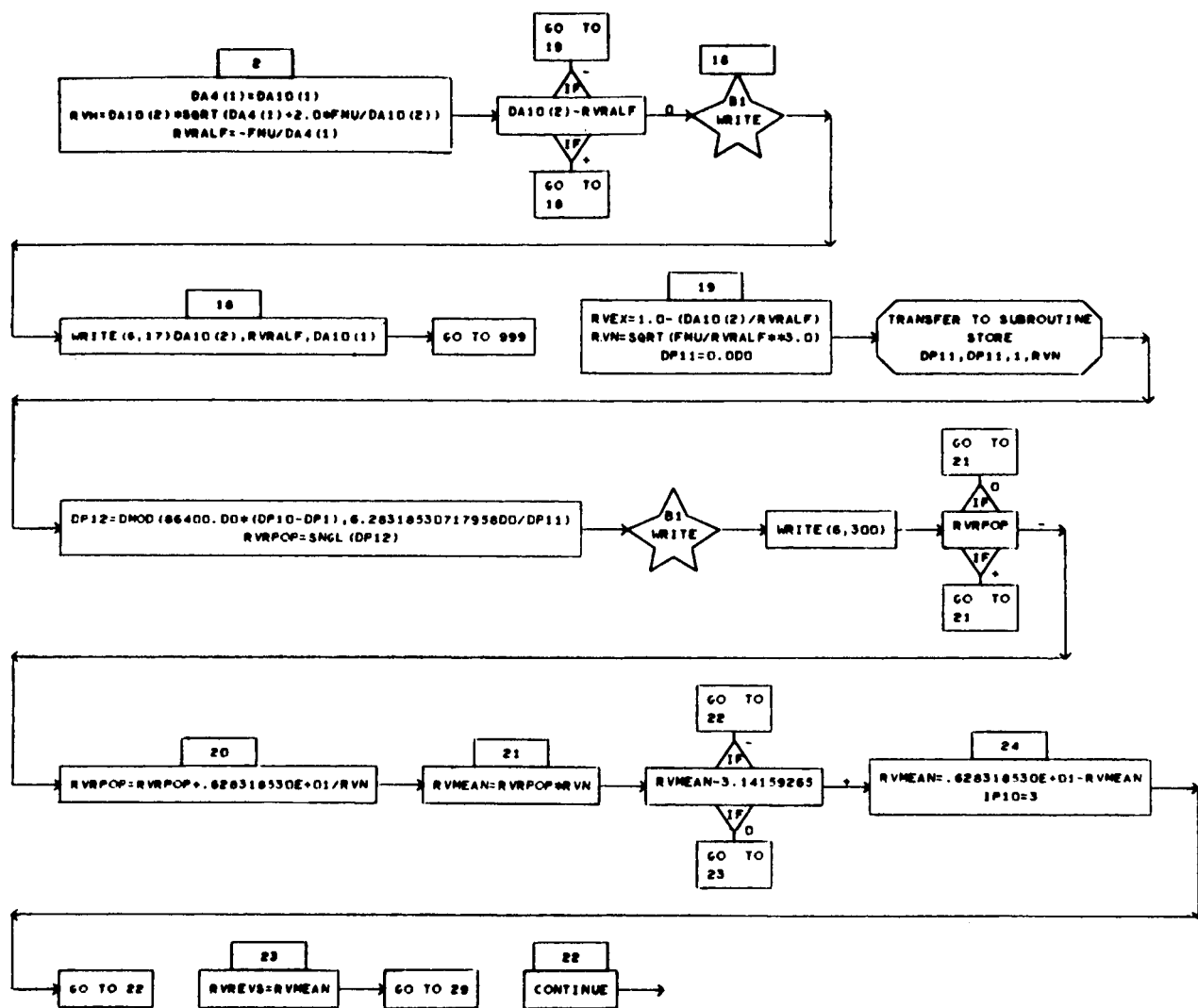


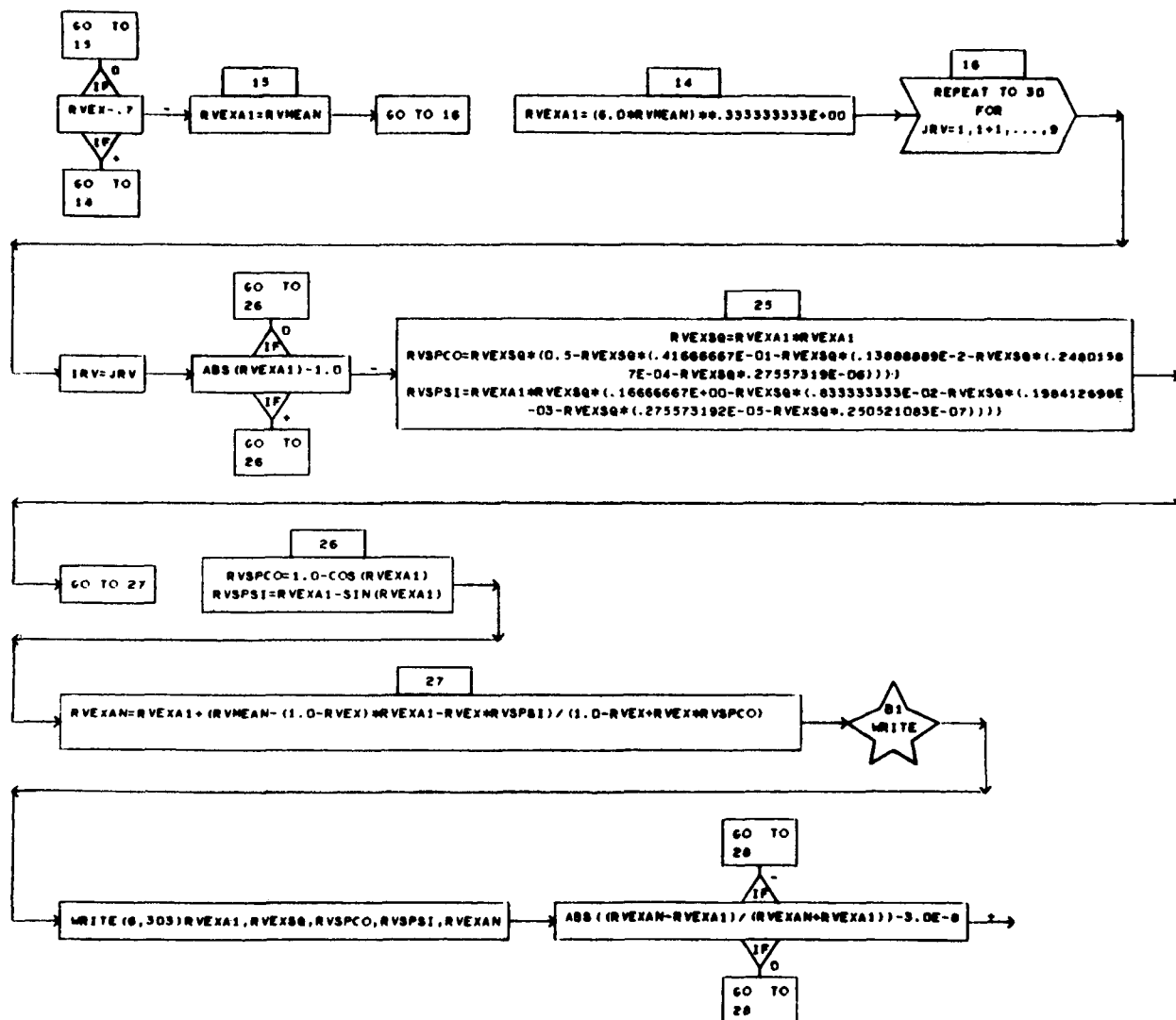
SUBROUTINE ROMBVU

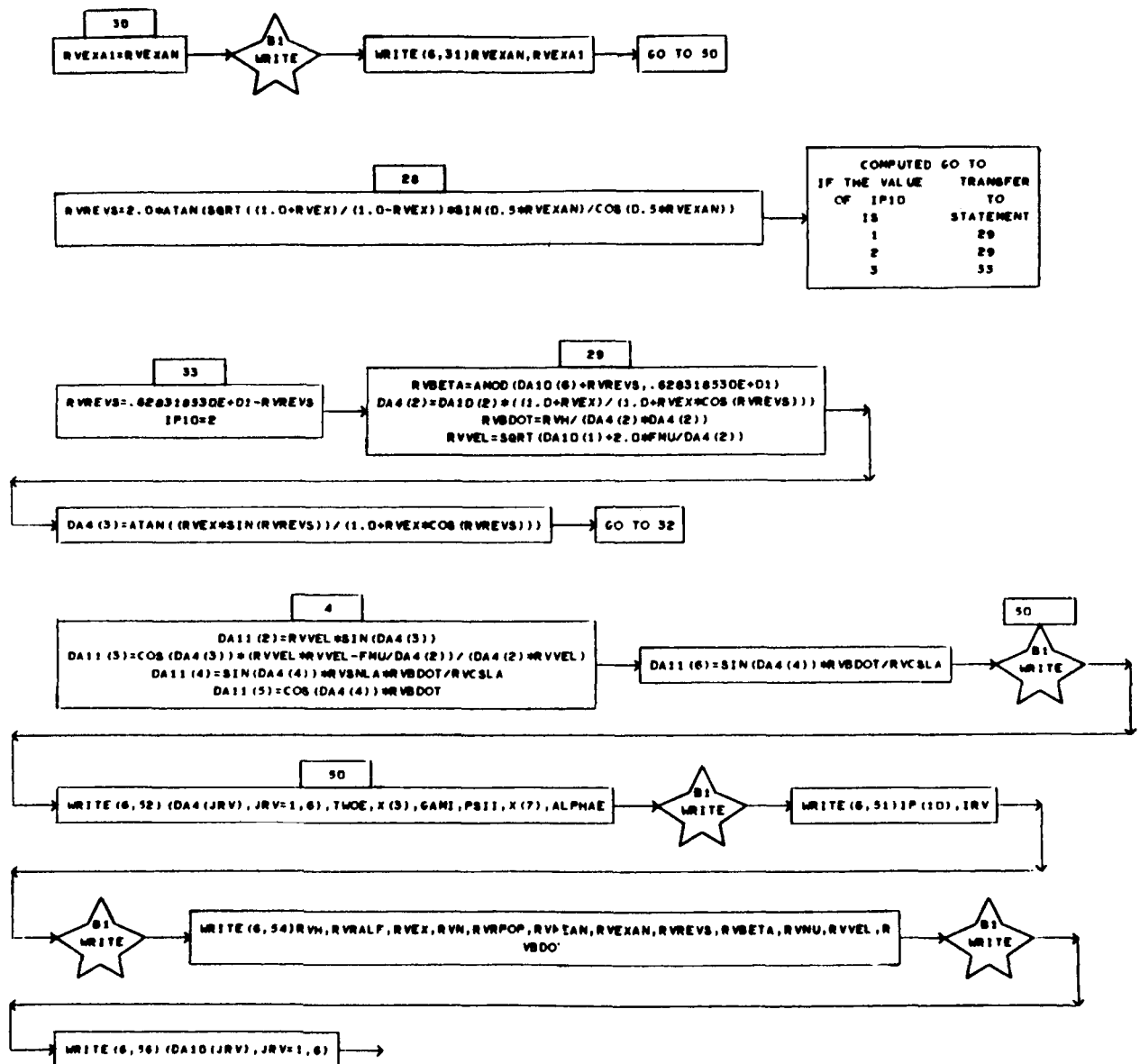


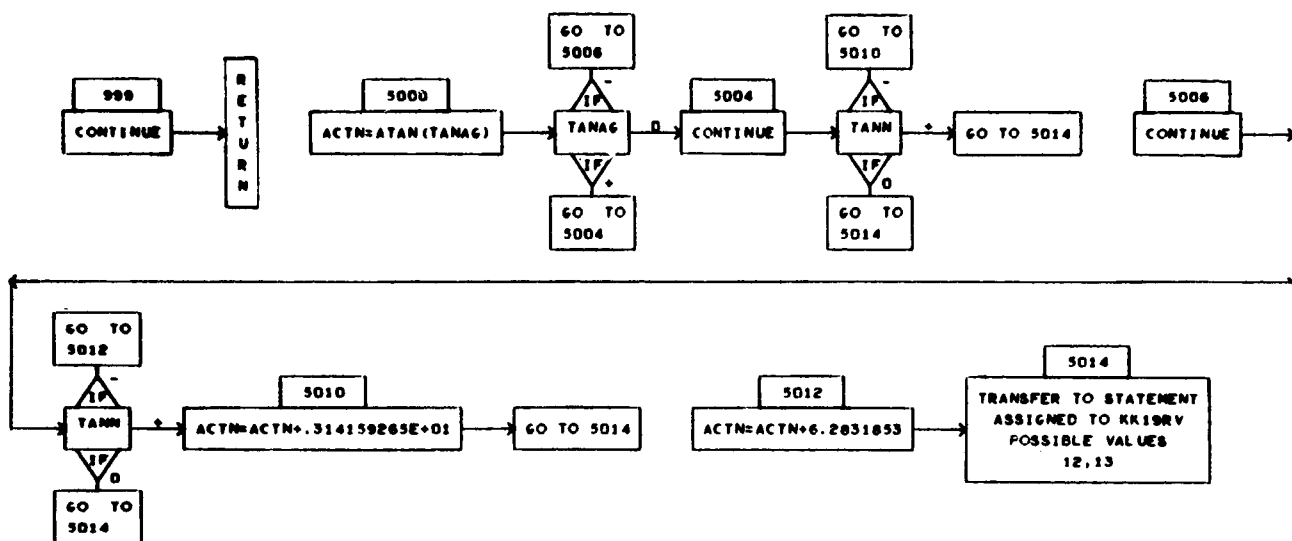












4. RENDEZVOUS CODING NOMENCLATURE

A listing of all variables appearing in the RONDVU subroutine, plus the new variables added to PRESTO

ACTN	Answer from 4-quadrant arctan routine
ALPHAE	Right ascension of booster's present position
ANEXCN	Eccentric anomaly of booster's position during coast (COAST)
ANMEAN	Mean anomaly of booster's position during coast (COAST)
BSIDOT	B vector * time derivative of terminal constraint (MEQ)
DA	Double precision form of A matrix (MEQ)
	<u>** Boosters stopping parameter and terminal constraints for RONDVU mission **</u>
DA 4(1) = 2E =	Twice the total energy of the orbit
DA 4(2) = r =	Instantaneous radius at the rendezvous time
DA 4(3) = γ_I =	Inertial flight path angle at the rendezvous time
DA 4(4) = ψ_I =	Inertial azimuth angle at the rendezvous time
DA 4(5) = λ =	Latitude angle at the rendezvous time
DA 4(6) = τ_I =	Inertial longitude angle at the rendezvous time
DA 4(7) = r_p =	Radius of perigee
DA 4(8)	Available
DA 5(2)	Fractional part of launch time in Space Age Date
	<u>** Orbit elements for the target **</u>
DA 10(1) = 2E =	Twice the total energy of the target's orbit
DA 10(2) = r_p =	Radius of perigee of the target's orbit

DA 10(3) = i = Inclination of the target's orbital plane
 DA 10(4) = Ω_e = Inertial longitude of the ascending node
 DA 10(5) Fractional part of perigee time in Space Age Days
 DA 10(6) = β_p = Argument of perigee measured in the target's plane
 DA 10(7) Integer part of perigee time in Space Age Days
 DA 10(8) Integer part of launch time in Space Age Days
 ** Target vector rates **
 DA 11(1) Rendezvous time in Space Age Days
 DA 11(2) = \dot{r} = Time derivative of target's radius
 DA 11(3) = $\dot{\gamma}_I$ = Time derivative of target's inertial flight path angle
 DA 11(4) = $\dot{\psi}_I$ = Time derivative of target's inertial azimuth angle
 DA 11(5) = $\dot{\lambda}$ = Time derivative of target's latitude angle
 DA 11(6) = $\dot{\tau}_I$ = Time derivative of target's inertial longitude angle
 DA 11(7) = 0.0
 DA 11(8) = 0.0

 DP 1 Double precision form of target's perigee time
 DP 3 Double precision form of DA 10(5), t_p fraction
 DP 4 Double precision form of DA 10(7), t_p integer
 DP 5 Double precision form of DA 5(2), TGO fraction
 DP 6 Double precision form of DA 10(8), TGO integer
 DP 7 Double precision form of tentative new fractional part of launch time in Space Age Days
 DP 8 Double precision form of current DTAU(8)
 DP 9 Double precision form of TIMCT
 DP 10 Double precision form of rendezvous time
 DP 11 Double precision form of rendezvous time integer for output
 DP 12 Double precision form of rendezvous time fraction for output

DTAU(8)	Change to be made in launch time of day
EXCENT	Eccentricity of booster's orbit during coast (COAST)
EXSQRT	Function used in relating true and eccentric anomalies (COAST)
FMU	μ , gravity constant
IP 10	Non-subscripted form of IP(10)
	<ul style="list-style-type: none"> = 0 RONDVU will not be called = 1 calls RONDVU for circular target orbit = 2 calls RONDVU for elliptic target orbit
IRV	Used as counter in RONDVU
JRV	Used as index in RONDVU
KK19RV	Return code for RONDVU assign go to statements
PARTL	Matrix of partial derivatives across a coast (COAST)
PTC	Partials of coast time with respect to trajectory variables (COAST)
PTCAZ1	Partial derivative of coast time with respect to azimuth (COAST)
PTCL1	Partial derivative of coast time with respect to latitude (COAST)
	<u>**RVxxx are RONDVU variables at the rendezvous time **</u>
RVBDOT	Targets true anomaly time derivative
RVBETA	Targets in plane range angle from ascending node
RVCBET	Cosine of RVBETA
RVCSIN	Cosine of inclination of target's orbit
RVCSLA	Cosine of target's latitude
RVEXA1	Target's eccentric anomaly, first estimate
RVEXAN	Target's eccentric anomaly
RVEX	Eccentricity of elliptic target orbit

RVEXSQ	RVEXA1 squared
RVH	Target's angular momentum
RVMEAN	Target's mean anomaly
RVN	Target's mean motion
RVNU	Target's longitude angle measured from ascending node
RVRALF	Semi-major axis of target's orbit
RVREVS	Target's true anomaly
RVS BET	Sine RVBETA
RVSNIN	Sine of inclination of target's orbit
RVSNLA	Sine of target's latitude
RVSPCO	Special cosine = $1. - \cos E_1$
RVSPSI	Special sine = $E_1 - \sin E_1$
RVRPOP	Required Part of Orbital Period
RVVEL	Target's inertial velocity at rendezvous time
ST 5(8)	Storage for DTAU(8), launch time adjustment
TANAG	Tangent of the angle
TANN	Numerator of TANAG
TG	Current time in space age date
TGO	Launch time in space age date
TIMCT	Time from launch including coasts
X(5)	Longitude

5. PROGRAM OPERATION

Data Input: Format and Discussion

DATA BLOCKS

Data required for the RONDVU version of PRESTO are grouped and input in data blocks exactly as in the basic PRESTO. The special RONDVU mission requirements are discussed below.

Data Block 4: Options

- (1) Lunar and planetary transfer missions are not available.
- (10) Triggers rendezvous computations. The RONDVU subroutine may be called for either a circular target orbit, or for an elliptic target orbit, or bypassed entirely. When bypassed, the program may be used to generate reference trajectories and corresponding target ephemeris data that would be the basis for parametric studies investigating perturbations in the target ephemeris.
- (20) Forces the perigee radius constraint to be imposed on the coast stage regardless of the normal tests: this is vital for successful computation of trajectories requiring violent maneuvers, as the perturbed target ephemeris trajectories mentioned above.

Data Block 5: Stage Sequence

Coast stage 9 must be used if the radius of perigee constraint is to

be applied. The circular park orbit option is not available.

Data Block 8: Terminal Constraints

RONDVU has been coded for the following sequence: stopping parameter = TWO E (Code 1), terminal constraints r , γ_I , \dot{v}_I , λ , τ_I (Codes 14, 7, 9, 10, 8). The longitude constraint (#8) has been converted to right ascension for the rendezvous mission. Provision has been made for up to eight terminal constraints, which is one more than required when the program adds the perigee radius constraint. If RONDVU is not called, the first four terminal constraints may be used for any of the available parameters. The fifth, if it is used, must be inertial longitude.

Data Block 15: Nominal Constants

(2) The gamma increment has been assigned the sign of gamma, and the value of .0002 is suggested.

Data Block 19: Terminal Constraint Magnitudes

RONDVU computes these magnitudes at the rendezvous time for each iteration. However, the magnitude of the stopping parameter TWO E must be input for the first case of every series of cases to a specific value of TWO E, since RONDVU is not called until the end of the initial nominal trajectory.

Data Block 20: Launch Time

The second word is the fractional part of the launch date expressed in Space Age Days, and must have the correct sign. For example, if TGO = -37.632, DA 5(2) = -0.632. (See Data Block 25.)

Data Block 23: State Variable Constraints

Minimum allowable altitude (feet) for coast stage 9, loaded in the third word, is used in the radius of perigee constraint (as in PRESTO).

Data Block 24: Nominal Range Angles

Accurate estimates for the nominal range angle traversed by coast stage 9 improve computational speed by reducing the terminal errors.

Data Block 25: Target Ephemeris

The orbit elements necessary to define the target vehicle orbit are specified. Angles are radian measure from $0 \rightarrow +2\pi$ only. The position of the target in the orbit is determined by specifying the time of passage through perigee, measured in Space Age Days. The eighth word is the whole part of the launch date. Two data word input for these dates (target perigee time and booster launch date) allows "double precision" accuracy. Since one second $\approx 10^{-5}$ day, a launch in 1970 requires TGO $\approx 365x.xxxx1$ for accuracy to within one second. Input of the whole part 365x. and the fractional part .xxxx1 separately gives even greater accuracy. The argument of perigee may be defined to be zero for convenience with circular orbits.

Data Block 26: Instantaneous Target Rates

Internal computation by RONDVU uses this data block for storage of the instantaneous state variable rates of the target at the rendezvous time. The quantities are stored in the same sequence as the terminal constraints (after the first) as defined in Section 5 for DA 11(x). Units are feet/second and radians/second.

Special Input Data

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
4	Options	24I1	IP(10) RONDVU flag = 0 bypass RONDVU = 1 circular target orbit = 2 elliptic target orbit IP(20) Perigee radius constraint flag = 0 normal tests = 1 impose constraint regardless
8	Terminal Constraints	9I3	For RONDVU, use stopping code 1. There are six terminal constraints including mass; use the list 14, 7, 9, 10, 8.
15	Nominal Constants	8E12.8	CT6(2) = .0002 is suggested
19	Terminal Constraint Magnitudes	8E12.8	DA4(1), Magnitude of the stopping parameter, must be input every time a <u>new</u> value is desired.
20	Initial Times	2E12.8	DA5(2) Fractional part of Launch Date (with correct sign) in Space Age Days (days from 1 Jan 1960, 0.0 hrs U.T.)
25	<u>Target</u> Ephemeris	8E12.8	DA10(1) $TWOE = (V_I^2 - 2\mu/R)$ DA10(2) Perigee radius, feet DA10(3) Inclination, radians DA10(4) Inertial longitude of ascending node, radians DA10(5) Decimal fraction of target's perigee time, S.A. Days DA10(6) Argument of perigee, radians DA10(7) Whole part of target's peri- gee time, S.A. Days DA10(8) Whole part of Launch Date, S.A. Days
28	Convergence Data	6E12.8	DA13(5) Velocity to start open-loop computations must occur before any coast or burn time adjust- able parameter is encountered.
29	Permitted Values of Terminal Constraint and Coast Perigee Constraint Deviations	7E12.8	DA14(1) List in the same order as the : constraints (with perigee DA14(7) radius last) Suggest 50,000 feet for distances and .008 radians for angles.

Output Format

RONDVU COMPUTATIONS

At the end of each forward trajectory, the RONDVU subroutine computes and outputs values for the pertinent times and state variables at the rendezvous time, which is defined as the time the booster achieves the desired value of the stopping parameter. Direct comparison is possible between the state variables of the target and those achieved by the booster; these are the terminal constraints that the program is operating on. Notice these are inertial quantities, i.e., γ_I , ψ_I , and inertial longitude, or more correctly, right ascension.

Target Perigee Time	Space Age Days
Booster Launch Time	= TGO, S.A. Days
Time from Launch,	
Including Coasts	= TIMCT, S.A. Days
Rendezvous Time	= TGO + TIMCT, S.A. Days
TWOE	$2E = V_I^2 - 2 \mu/R$, (ft/sec) ²
Radius	r , feet
Flight Path Angle	γ_I , radians
Azimuth	ψ_I , radians
Latitude	λ , radians
Longitude	α_e , radians

Subtraction of the booster variable from the target variable gives the terminal constraint error ($d\psi$), which, when multiplied by the proper DPDPSEI from the backward trajectory, gives the change in final weight associated with that error.

In addition, other target RONDVU variables are output, although most are output as zero for circular orbits (Option 1). For elliptic

orbits (Option 2), all quantities are output: they are defined in Section 5 - Nomenclature. Units are as used internally in the program; feet, seconds, radians, and radians/second for angular rates (RVN and RVBDOT).

The input target ephemeris is also output, as are the achieved values of the booster's orbit elements at injection.

Trouble Shooting

Listed in this section are some of the peculiar difficulties that can be encountered using the RONDVU subroutine. As in PRESTO a great many of the troubles result from erroneous or omitted input data. Other troubles that may be encountered and their solutions are listed below.

- o Failure to input TWOE initially or whenever a new value is desired.
- o Usage of the coast stage will generally require calling the radius-of-perigee constraint. For small perturbations on a reference target ephemeris, the internal tests may be sufficient to trigger this constraint. However, for large target ephemeris perturbations, this constraint must be imposed through the use of Option 20.
- o Poor input nominals, especially in terms of input THETA, CHI, weights, and burn times. The RONDVU subroutine when used for trajectories with large perturbations from a reference target ephemeris becomes very sensitive to inputs. For example, a maximum payload trajectory might have only two to four seconds of burn time after the coast to injection into orbit. Since the trajectory is terminated by the stopping parameter, if the first burn is too long, the stopping parameter will be achieved prior to the coast stage. If the burn time is too short, and perhaps only by three quarters of a second, insufficient velocity will be available at the start of the coast stage to

achieve the desired terminal altitude.

- o Because of the repetitive nature of the timing of the problem, there are many solutions for a given ephemeris. The RONDVU version of PRESTO, as the basic PRESTO, will only seek a local optimum. This implies that the user has an excellent idea of what region he wants to be in for the result.
- o The above arguments also imply that only small step sizes should be taken in developing a parametric perturbation analysis from a nominal. A small change in launch time produces a large change in the target's position forcing changes in the trajectory that are beyond the booster's capability.

6. EQUATIONS

RONDVU EQUATIONS

A specified target ephemeris of $2E$, r_p , i , Ω_e , t_p and β_p is combined with a rendezvous time TG to evaluate the target's instantaneous position in its orbit, expressed in terms of PRESTO's state variables, along with their time derivatives.

Circular Orbit

$$2E = -\mu/r_p$$

$$n = (\mu/r^3)^{\frac{1}{2}}$$

$$\theta' = (TG - t_p) \cdot n$$

$$\beta = \beta_p + \theta'$$

$$r = r_p$$

$$\gamma_I = 0$$

$$\tau_I = \alpha_e = 99.659967^\circ + 0.98564743 \cdot TG + 360.0 \cdot (TG \text{ fraction}) + \tau$$

$$\tan \psi_I = \cos i / (\sin i - \cos \beta)$$

Elliptic Orbit

$$2E = 2E$$

$$RVH = r_p \cdot (2E + 2\mu/r_p)^{\frac{1}{2}}$$

$$RV-r_\alpha = -\mu/2E$$

$$e = 1 - (r_p/r_\alpha)$$

$$n = (\mu/r_\alpha^3)^{\frac{1}{2}}$$

$$RPOP = TG - t_p$$

$$M = (TG - t_p) \cdot n$$

$$E' = M - e \sin E'$$

$$\tan \left(\frac{\theta'}{2} \right) = \left(\frac{1+e}{1-e} \right) \tan \left(\frac{E'}{2} \right)$$

$$\beta = \beta_p + \theta'$$

$$r = \frac{r_p (1+e)}{1+e \cos \theta'}$$

$$\dot{\beta} = H/r^2$$

$$v_I = (2E + 2\mu/r)^{\frac{1}{2}}$$

$$\tan \gamma_I = \frac{e \sin \theta'}{1+e \cos \theta'}$$

$$\sin \lambda = \sin i \cdot \sin \beta$$

$$\tan v = (\cos i \cdot \sin \beta) / \cos \beta$$

$$\tau_I = v + \Omega_e$$

Circular Orbit

$$\dot{\psi}_I = \sin \psi_I \cdot \sin \lambda \cdot n / \cos \lambda$$

$$\dot{\lambda} = \cos \psi_I \cdot n$$

$$\tau_I = \sin \psi_I \cdot n / \cos \lambda$$

Elliptic Orbit

$$\dot{r} = V_I \sin \gamma_I$$

$$\dot{\gamma}_I = \cos \gamma_I \cdot (V_I^2 - \mu/r) / (r V_I)$$

$$\dot{\psi}_I = \sin \psi_I \cdot \sin \lambda \cdot \dot{\beta} / \cos \lambda$$

$$\dot{\lambda} = \cos \psi_I \cdot \dot{\beta}$$

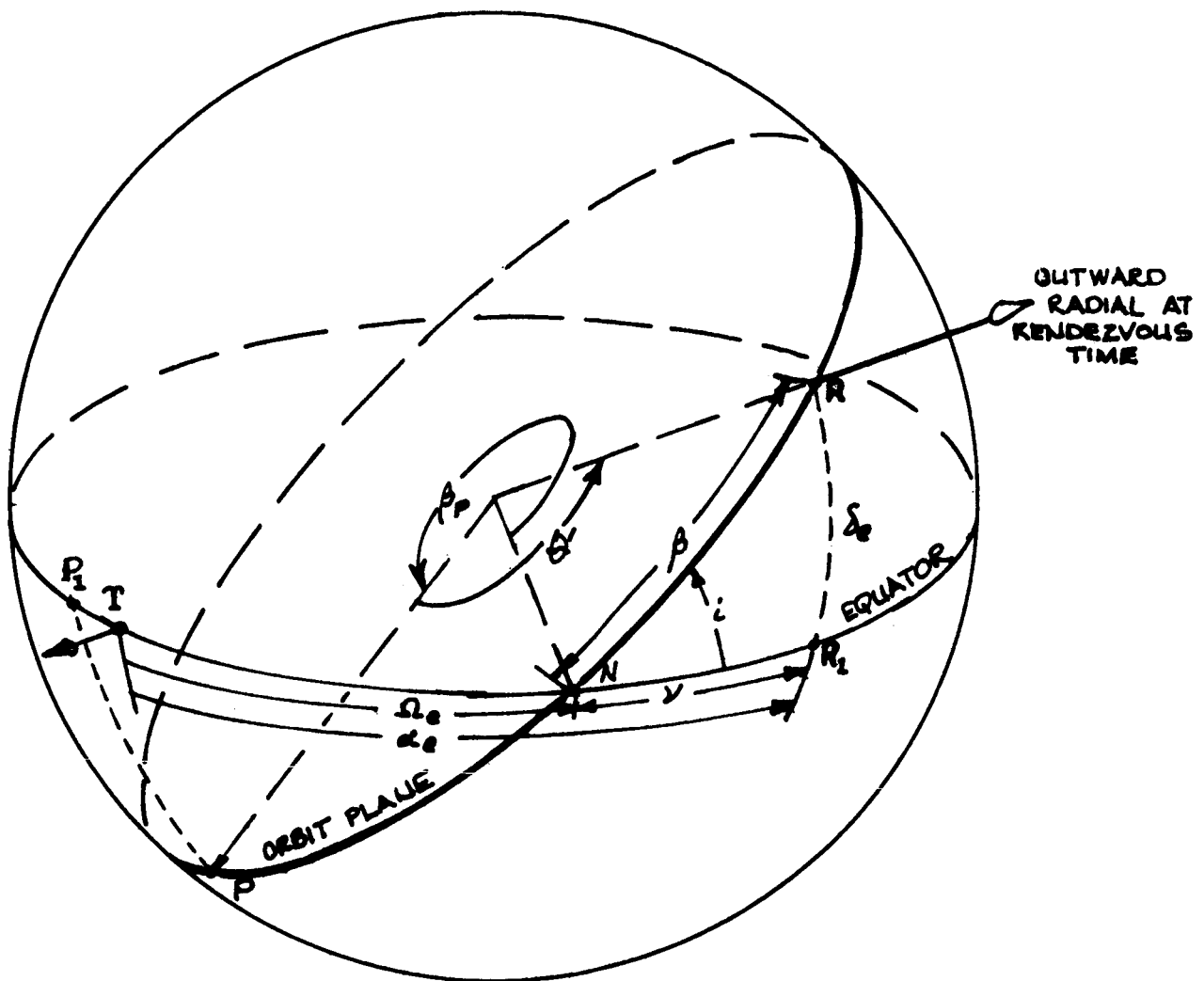
$$\dot{\tau}_I = \sin \psi_I \cdot \dot{\beta} / \cos \lambda$$

LIST OF SYMBOLS INTRODUCED FOR RONDVU

e	Eccentricity of target orbit
E'	Eccentric anomaly
M	Mean anomaly
n	Mean angular motion
RPOP	Required Part of Orbital Period, seconds
RVH	Angular momentum of target's orbit
RV-r _α	Semi-major axis of target's orbit
t _p	Target's perigee time in Space Age Date
θ'	True anomaly

The remaining nomenclature is identical to that of PRESTO, although when used in the RONDVU subroutine it applies to the target, not the booster.

IN INERTIALLY-ORIENTED EQUATORIAL SYSTEM



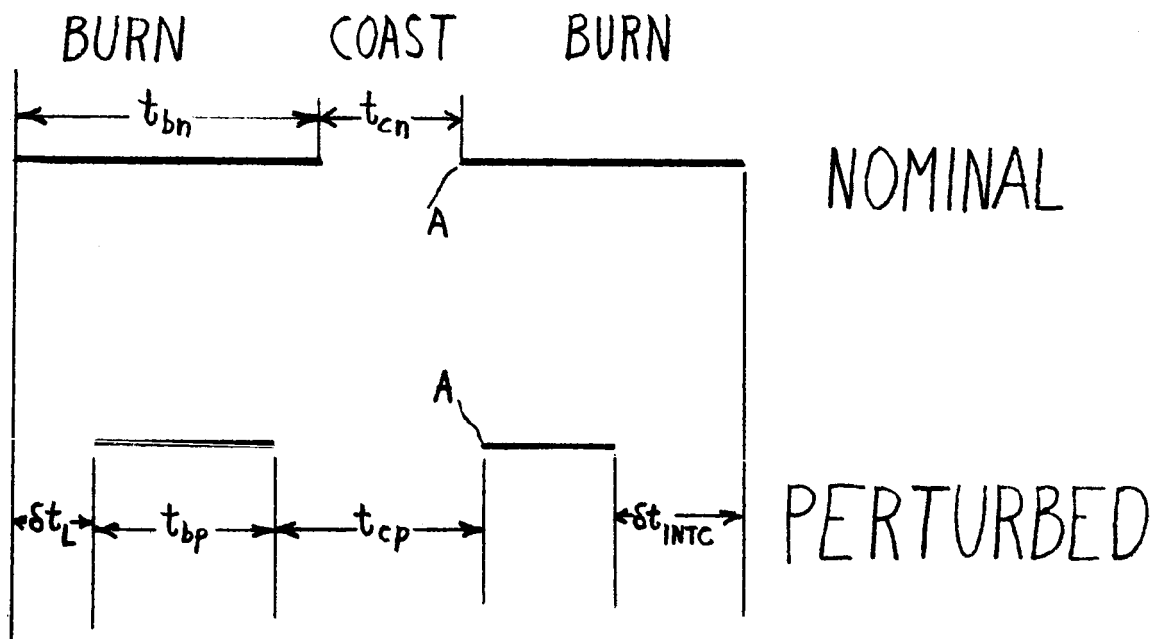
Outward Radial Direction varies with time as the target travels in its orbit plane.

R	Target's position at rendezvous time	β	In-plane angle from ascending node
R_1	Equatorial projection of R	β_p	Argument of perigee
P	Perigee of target's orbit	i	Inclination of target's orbit plane
P_1	Equatorial projection of P	δ_e	Declination of Outward Radial
γ	Vernal equinox	ν	Inertial longitude angle from ascending node
θ'	True anomaly	Ω_e	Longitude of N
α_e	Right ascension of Outward Radial	N	Ascending node of the orbit plane

7. DERIVATION OF OPTIMIZATION EQUATIONS AND PROGRAMMING FOR RENDEZVOUS OPTION

The distinguishing feature of the rendezvous problem is that the terminal constraints are the position and velocity of a moving target and are therefore functions of time. In order to satisfy a time-dependent terminal constraint, several changes must be made in the PRESTO equations.

The determination of the influence of adjustable launch, coast, and burn times on terminal constraints does not make use of an absolute time scale. Consider the two trajectories sketched below.



where

t_{bn} is the length of the adjustable burn on the nominal trajectory

t_{bp} is the length of the adjustable burn on the perturbed trajectory

t_{cn} is the length of the adjustable coast on the nominal trajectory

t_{cp} is the length of the adjustable coast on the perturbed trajectory

δt_L is the change in launch time

δt_{INTC} is the change in the burnout or intercept time

The change in intercept time will be the sum of the changes in the adjustable parameters plus the change in time of the final burn stage. It is important to note that the perturbed trajectory at point A corresponds to the nominal trajectory at point A regardless of the fact that the absolute times are different. The effect of the change in burn time from A to the end of the trajectory is accounted for when the adjoint variables are initialized. The effect of all changes in time up to point A must be accounted for by adjusting the S associated with the adjustable parameters.

The basic perturbation equation for the rendezvous problem can be written as

$$d\psi + \dot{\psi}_{target} [\delta t_{launch} + \delta t_{burn} + \delta t_{coast}] = \lambda \delta x + \int_{t_f}^t \Lambda \delta \alpha dt + S \delta \tau \quad (1)$$

where δt_{burn} refers to the change in an adjustable burn stage and does not include the final burn,

and $\dot{\psi}_{target}$ is the vector of target rates evaluated at the nominal intercept time.

The presence of the $\dot{\psi}_{target}$ term in Eq. (1) corrects the desired change in the terminal constraints ($d\psi$) for any changes made in the

adjustable parameters. Eq. (1) may be rewritten as

$$d\psi = \lambda \delta x + \int_{t_f}^t \Lambda \delta \alpha dt + (S - \dot{\psi}_{\text{target}}) \delta \tau$$

The columns of the S matrix associated with adjustable launch, coast, and burn times are adjusted by subtracting $\dot{\psi}_{\text{target}}$ from them. This is done in the TRAJ subroutine at the place where S is computed.

On a forward trajectory, after the launch time has been changed, the $d\psi$ vector must be changed to include the additional term $\dot{\psi}_{\text{target}} \delta t_L$. However, during the closed-loop portion of a forward trajectory, $d\psi$ is not used. It has already been incorporated into B_{12}^* and B_{22}^* on the previous backward run. (See page 8-18 in PRESTO manual.) It is necessary to store two additional quantities during the closed-loop portion of a backward run.

Write the control vector $\delta \alpha$ during closed-loop operation as

$$\delta \alpha = B d \psi - D \delta x \quad (2)$$

where $B = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1JC} \\ B_{21} & B_{22} & \dots & B_{2JC} \end{bmatrix}$

After the launch time adjustment, $d\psi$ is replaced by $d\psi + \dot{\psi}_{\text{target}} \delta t_L$.

Substitute this into the expression for $\delta \alpha$ to obtain

$$\delta \alpha = B d \psi + B \dot{\psi}_{\text{target}} \delta t_L - D \delta x \quad (3)$$

Expanding Eq. (3), one obtains

$$\begin{bmatrix} \delta\eta \\ \delta\chi \end{bmatrix} = \begin{bmatrix} B_{11} d\psi_1 + \sum_{j=2}^{JC} B_{1j} d\psi_j + \left(\sum_{j=2}^{JC} B_{1j} \dot{\psi}_{j_{\text{target}}} \right) \delta t_L \\ B_{21} d\psi_1 + \sum_{j=2}^{JC} B_{2j} d\psi_j + \left(\sum_{j=2}^{JC} B_{2j} \dot{\psi}_{j_{\text{target}}} \right) \delta t_L \end{bmatrix} - D\delta x \quad (4)$$

The quantities to be stored on the backward run during closed-loop operation are

$$\sum_{j=2}^{JC} B_{1j} \dot{\psi}_{j_{\text{target}}}$$

and

$$\sum_{j=2}^{JC} B_{2j} \dot{\psi}_{j_{\text{target}}}$$

The two sums are computed in MEQ and are stored in BSIDOT (MB1) and BSIDOT (MB2), respectively.

On forward runs, during closed-loop operation, BSIDOT is multiplied by δt_L and is added to the equations for $\delta\eta$ and $\delta\chi$. This is done in PCAL. When the K vector is computed in PCAL at the t_9 point, $\dot{\psi}_{\text{target}} \delta t_L$ is added to $d\psi$.

The initialization of the adjoint variables must also be changed to take into account the time-dependent terminal conditions. Let $r(x)$ be the desired stopping condition and let T be the unknown time at which the stopping condition is satisfied. t_f is the nominal stopping time. One may write

$$d\psi|_T = d\psi|_{t_f} + (\dot{\psi} - \dot{\psi}_{\text{target}}) \Big|_{t_f} \delta t \quad (5)$$

where it is understood that δt refers only to the change in time of the final burn. Similarly,

$$d\lambda|_T = d\lambda|_{t_f} + \dot{\lambda}|_{t_f} \delta t \quad (6)$$

The stopping condition is satisfied when $d\lambda|_T = 0$. Solve for δt in Eq. (6) and substitute in Eq. (5). The result is

$$d\psi|_T = d\psi|_{t_f} - \left. \frac{\dot{\psi} - \dot{\psi}_{\text{target}}}{\dot{\lambda}} \right|_{t_f} d\lambda|_{t_f} \quad (7)$$

or

$$d\psi|_T = \left[\frac{\partial \psi}{\partial x} - \frac{\dot{\psi} - \dot{\psi}_{\text{target}}}{\dot{\lambda}} \frac{\partial \lambda}{\partial x} \right]_{t_f} \delta x \quad (8)$$

From Eq. (8), it is seen that the target rate is subtracted from the terminal constraint rate before computing the initial conditions on the adjoint variables. This is done in the ICS subroutine.

Change in Coast Radius-of-Perigee Constraint

The purpose of the radius-of-perigee constraint is to insure that the altitude of the trajectory during coast remains above a minimum altitude. If the coast does not contain the perigee, however, application of this constraint will cause the coast altitude to exceed the minimum by an amount which depends on the geometry of the orbit. The payload would then be unnecessarily penalized.

The following procedure has been adopted to avoid this problem. If the flight path angle at coast injection is positive, the minimum coast

altitude will, in general, be the altitude at the start of the coast. If this altitude violates the minimum altitude constraint, it will be constrained to the desired value in place of constraining the radius of perigee. If the flight path angle at the start of coast is negative, the coast will go through the perigee and the radius-of-perigee constraint will be applied. During the iterative procedure, the program can switch back and forth between the two constraints.

There is one exception. If the final altitude is lower than the altitude at the start of the coast, the constraint will be applied to the altitude at the start of the coast regardless of the sign of the flight path angle. The radius-of-perigee constraint does not work well under these conditions.

A switch, IS(40), is set to +1 or -1 depending on the sign of the flight path angle at the start of the coast. This is done in the coast section of the TRAJ subroutine. If the altitude constraint is violated, this switch is used in the ICS subroutine to determine the set of adjoint variables that will be initialized.

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